Statistics 116 - Fall 2004
Theory of Probability
Practice Midterm # 1

SHOW (AND BRIEFLY EXPLAIN) ALL OF YOUR WORK.

CALCULATORS ARE PERMITTED FOR NUMERICAL CALCULATIONS ONLY.

Instructions: Answer 4 out of 5 questions. Clearly mark which 4 questions you decide to answer. If you do not clearly indicate which 4 are to be counted, your mark will be based on 5 instead of 4 questions, there are no bonus points. All questions have equal weight.

Q. 1) The second Earl of Yarborough is reported to have bet at odds of 1000 to 1 that a hand of 13 cards (out of a deck of 52) would contain at least one card that is ten or higher. (By *ten or higher* we mean that a card is either a ten, jack, queen, king or ace.) What is the probability that the Earl wins this bet?

Q. 2) A deck of 52 cards is shuffled and then divided into two halves of 26 cards each. A card is drawn from one of the halves; it turns out to be an ace. The ace is then placed in the second half-deck. This half is then shuffled, and a card is drawn from it. Compute the probability that the card drawn is an ace.

Q. 3) Let P_n denote the probability that n independent trials with probability of success p result in an even number of successes (0 being considered an even number).

(a) Show that

$$P_n = p \cdot (1 - P_{n-1}) + (1 - p) \cdot P_{n-1}, \qquad n \geq 1.$$

(b) Use this to prove, by induction, that

$$P_n = \frac{1 + (1 - 2p)^n}{2}.$$

Q. 4) In the game craps, two dice are rolled and certain bets are made on the outcome of these rolls. A casino has accused a particular gambler of secretly replacing the usual fair dice with two identical weighted dice, where the weights are suspected to be $(1/4, 1/4, 1/6, 1/6, 1/12, 1/12)$ (i.e. the probability that a "1" is rolled is $1/4$ and the probability that a "4" is rolled is $1/6$) as opposed to the usual weights of $(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)$. You, an inspector, are sent in to investigate.

(a) If the weighted dice (with weights as above) are rolled, what is the probability that a pair is rolled?

(b) Suppose that, from experience, you know there is a 5% probability that the casino's allegation is true. To test the claim, you decide to roll the two possibly weighted dice 20 times and count the number of pairs rolled. If you rolled 6 pairs out of 20 possible, what is the conditional probability that the dice you rolled were weighted?

Q. 5) A family has $n \geq 1$ children with probability $\alpha \cdot p^n$ where $\alpha \leq (1 - p)/p$.

(a) What proportion of families have no children?

(b) Suppose each child, independently of the others, is equally likely to be a boy or a girl. What proportion of families consists of k boys (and any number of girls)?