

STATISTICS 116 MIDTERM EXAM

April 20, 2005

NAME: _____

- You have 50 minutes to complete the exam.
- There are 5 questions. ANSWER ALL QUESTIONS.
- Each question is worth 20 points.
- Except for question 1, show all your work and justify your answers. You may receive no credit if you don't.
- If you use more than one page for your answer, say so, so we don't miss your work.
- You may use a calculator; no other aids are allowed.

GOOD LUCK!

For graders: 1 _____ 2 _____ 3 _____ 4 _____ 5 _____

TOTAL GRADE: _____

1. For each of the following statements, say whether it is true or false. There is no need to justify your answer. Each question is worth 4 points.

(a) For any events E and F , $P(E \cap F) = P(E) \cdot P(E|F)$.

(b) If the events A and B are mutually exclusive, then $P(A) + P(B) \leq 1$.

(c) With a fair coin, the chance of getting exactly 5 heads in 10 tosses is the same as the chance of getting exactly 50 heads in 100 tosses.

(d) With a fair coin, the chance of getting at least 3 heads in 5 tosses is the same as the chance of getting at least 4 heads in 7 tosses.

(e) If X is uniform on $[0,2]$, then $P(X = 1) = 0$.

2. Let X and Y be uniform over $[0,1]$.

(a) (5 pts) What is the probability that one of the two numbers X, Y is at least twice as large as the other one?

(b) (10 pts) Let $Z = X \cdot Y$. Compute the CDF and the density of Z .

(c) (5 pts) Find a number t such that $P(Z < t) = 2t$.

3. A poker hand (5 cards) is dealt from an ordinary deck of 52 cards. Find the probability that:

- (a) (5 pts) There is at least one ace.
- (b) (15 pts) The hand is three-of-a-kind.

NOTE: Three-of-a-kind means that three cards have the same value and the other two have different values. For example, AAA57 is three-of-a-kind, but AAAA7 and AAA55 are not.

4. Toss a fair coin 6 times. Each time you get a head, you move one yard to the left; each time you get a tail, you move one yard to the right. Let X be the distance to your starting point (after 6 tosses).

(a) (5 pts) Plot the distribution of X .

(b) (5 pts) If before you started you were asked to guess X , what would be your best guess? Why?

(c) (5 pts) If you know that the first three tosses were heads, what is your best guess for X now?

(d) (5 pts) Let E be the event that $X = 0$, and F be the event that the third toss was a head. Are E and F independent?

5. (20 pts) A patient goes to the doctor for a routine checkup. The doctor performs a series of standard tests, including one for a particular type of cancer. The test is not perfect, but it is pretty good: if you have the disease you test positive 99% of the time, while if you don't have the disease you test negative 91% of the time. The doctor expects the test to be negative, since this kind of disease is rare (about 1 in 100 for persons with this patient's profile). However, the test comes out positive. Conditional on this, what is the probability that the patient has the disease? Should he/she be worried?