

STATISTICS 200 Practice Final, March 2004

1. Hans and Franz want to play a die game. They suspect that the die they have is not fair, but for the particular game they have in mind it would be sufficient if opposite faces would stand the same chance of showing up if the die is tossed. That condition can be expressed by $Prob(1) = Prob(6) = p_1$, $Prob(2) = Prob(5) = p_2$ and $Prob(3) = Prob(4) = p_3$, for some nonnegative p_i 's that sum up to one-half. Hans and Franz roll the die one hundred times and observe the following outcomes:

1	2	3	4	5	6
6	12	20	30	18	14

- Test for a fair die, using $\alpha = 0.05$.
- Find the maximum likelihood estimators of p_1 and p_2 , assuming the model that assumes opposite faces have the same chance is correct.
- Test the null hypothesis that opposite faces have the same chance, also using $\alpha = 0.05$.

2. Suppose X_1, \dots, X_n are independent and identically distributed according to the Poisson distribution with mean θ (and variance θ as well).

(i). To test the null hypothesis that the mean θ is equal to one against the alternative hypothesis that the mean is greater than one, you consider a rejection region of the form $\bar{X}_n \geq c_n$, where $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$ is the usual sample mean and c_n is a sequence of constants possibly depending on the sample size n . How should c_n be chosen so that the probability of a type 1 error is no greater than one percent?

(ii). If the actual population mean is 1.21, how large must the sample size n be so that your test in part (i) has probability of a type 2 error no bigger than 5 percent?

3. In an experiment by Collins et al., one member of a pair of corn seedlings was treated by a small electric current; the other was untreated. After a period of growth, the differences in elongation (treated minus untreated) are (in mm.): 6.0, 1.3, 10.2, 23.9, 3.1, 6.8, -1.5, -14.7, -3.3, 11.1. Do these data support the hypothesis that electric current treatment affects elongation?

4. Suppose X_1, \dots, X_n are i.i.d. with density function

$$f(x, \theta) = \exp[-(x - \theta)]$$

for $x \geq \theta$ and $f(x, \theta) = 0$ otherwise. Here, the unknown parameter θ can be any real number.

- Find a method of moments estimator $\hat{\theta}_{MM}$.
- Compute the mean squared error of your estimator; that is, compute $E_\theta[(\hat{\theta}_{MM} - \theta)^2]$.

- (iii). Compute the maximum likelihood estimator for θ .
 - (iv). Find the distribution of the maximum likelihood estimator.
 - (v). Compute the mean squared error of the maximum likelihood estimator.
5. In a 1970 San Francisco court case, the issue of whether pin ball is a game of skill or chance was on trial. The following experiment was actually performed. A “skilled” player played 1000 games and won 100 games. (If you are not familiar with pin ball, think of a game where a player plays against the pin ball machine, and the player wins or loses.) A “nonskilled” player was asked only to set the ball in play and then accumulate a score without using any skill (or even the flippers!). The nonskilled player won 50 of the 1000 games. Is the difference due to chance? In the end, the judge was concerned because the chance of winning even for the skilled players seems small. The issue in the case was whether or not the observed differences was actually due to skill. As an expert witness, what would you tell the judge?
6. Let $(X_1, Y_1), \dots, (X_n, Y_n)$ be i.i.d. pairs of random variables. Further suppose X_i is independent of Y_i (so all the variables in sight are mutually independent). Assume the i th pair has mean μ_i (possibly different), but all the variables have a common unknown variance σ^2 . Finally, assume the variables all have a normal distribution; remember the normal density with mean μ and variance σ^2 is

$$f(x) = (2\pi\sigma^2)^{-1/2} \exp[-(x - \mu)^2 / (2\sigma^2)].$$

- (i). Find the maximum likelihood estimator of σ^2 .
- (ii). Compute the expected value of the estimator.
- (iii). Is the estimator consistent?