STATISTICS 200
MIDTERM
February 15, 2006

Name:

Instructions:

1. Print your name.
2. Each of the three problems is worth 10 points.
3. You must show your work to get full credit.
4. Do not turn the page until the signal is given.
5. Draw a box around your final answers.
6. Good luck!

Points:

Problem 1:

Problem 2:

Problem 3:

Total:

1. Let $X_1, \ldots, X_n$ be an i.i.d. sample from the uniform distribution on $(-\theta, \theta)$.

(i). Find a method of moments estimator of θ.

(ii). What is the approximate variance of your estimator?

(iii). Is your estimator unbiased for θ? If not, is the bias positive or negative, and why?

2. Out of 50,000,000 instant winner lottery tickets, the proportion of winning tickets is p. Each day, for 20 consecutive days, a bettor purchased tickets, one at a time, until a winning ticket was purchased. The numbers of losing tickets that were purchased each day before the winning ticket was purchased are the following: 2, 33, 6, 3, 18, 1, 0, 18, 52, 1, 21, 3, 18, 10, 6, 0, 1, 20, 14, 15.

(i). By making reasonable assumptions, find the maximum likelihood estimator for p based on these data.

(ii). Construct an approximate 95 percent confidence interval for p.

3. Suppose $Y_1, \ldots, Y_n$ is a sample of n independent observations having cumulative distribution function $F(y) = Prob(Y_i \leq y)$. Consider estimating $F(y)$ by $\hat{F}_n(y)$, where $\hat{F}_n(y)$ is the proportion of observations in the sample less than or equal to y.

(i) If F is the c.d.f. of the uniform distribution on (0,1) and $y \in [0, 1]$, what is the variance of $\hat{F}_n(y)$?

(ii) Again assume F is the c.d.f. of the uniform distribution on (0,1). If $x < y$, what is the covariance between $\hat{F}_n(x)$ and $\hat{F}_n(y)$?

(iii) For general F, what is the variance of $\hat{F}_n(y)$ in terms of F and n?