STATISTICS 200
MIDTERM
February 11, 2004

Name:

Instructions:

1. Print your name.
2. Each of the two problems is worth 20 points.
3. You must show your work to get full credit.
4. Do not turn the page until the signal is given.
5. Draw a box around your final answers.
6. Good luck!

Points:

Problem 1:

Problem 2:

Total:

1. **Problem 1:** Let $X_1, \ldots, X_n$ be an i.i.d. sample from a distribution with density function

$$f(x;\lambda) = \frac{x}{\lambda}\exp[-x^2/2\lambda],$$

where the parameter λ is known to be positive and $x > 0$ as well.

(i). *(5 points)* Find the MLE for λ.

(ii). *(5 points)* Calculate $E_\lambda(X_i^2)$. *Hint:* Do not integrate by parts; instead, use the fact that the derivative with respect to λ of the loglikelihood has mean 0.

(iii) *(5 points)* Calculate $I(\lambda)$, the Fisher Information one observation contains about λ. *Hint: Use the second derivative of the loglikelihood.*

(iv). *(5 points)* Is the MLE a sufficient statistic?

Problem 2: *(How to ask embarrassing questions)* You are asked to estimate the percentage of students on campus who have ever cheated, but you do not want to introduce a bias into your sample due to some students' reluctance to tell the truth. Tell each person sampled to secretly flip a fair coin. If the person gets a head, the person answers the question: *Have you ever cheated?* Otherwise, the person answers the innocuous question: *Were you born in California?* The information you get from each response is a yes or no (and not the outcome of the coin toss). Furthermore, you know from admissions records that the proportion of students on campus born in California is .6.

(i). *(5 points)* Let θ denote the proportion of students on campus who have ever cheated. As a function of θ, what is the probability that someone sampled responds with a yes?

(ii). *(5 points)* Let X denote the number of yes responses in your sample of size n. Suggest an estimator of θ. (Your answer should be a function of X and n.)

(iii). *(5 points)* Find the variance of the estimator.

(iv). *(5 points)* Find an approximate 95 percent confidence interval for θ based on a sample of 100 with half responding yes.