

Statistics 200, Homework 3

Due Tuesday, January 27, 2009

The first five problems are from the text: 5.1, 5.16, 5.21, 5.24, 7.1. Note that in Problem 7.1, the 2 observations are sampled *without replacement*.

6. Suppose $Y_n \xrightarrow{d} y$, where y is a constant. Show that $Y_n \xrightarrow{P} y$.

7. Let X be binomial based on n trials and success probability p . Let $\hat{p}_n = X/n$. Also, let $z_{1-\alpha}$ denote the $1 - \alpha$ quantile of the standard normal distribution $N(0, 1)$. From class we know that

$$\frac{n^{1/2}(\hat{p}_n - p)}{[\hat{p}_n(1 - \hat{p}_n)]^{1/2}} \xrightarrow{d} N(0, 1) , \quad (1)$$

which leads to the confidence interval statement that

$$P\{p \in \hat{I}_n\} \rightarrow 1 - \alpha , \quad (2)$$

where \hat{I}_n is the interval

$$\hat{I}_n = \hat{p}_n \pm z_{1-\frac{\alpha}{2}} n^{-1/2} [\hat{p}_n(1 - \hat{p}_n)]^{1/2} . \quad (3)$$

Instead of this interval, first start with the approximation obtained from the limit result

$$\frac{n^{1/2}(\hat{p}_n - p)}{[p(1 - p)]^{1/2}} \xrightarrow{d} N(0, 1) , \quad (4)$$

which implies

$$P\{-z_{1-\frac{\alpha}{2}} \leq \frac{n^{1/2}(\hat{p}_n - p)}{[p(1 - p)]^{1/2}} \leq z_{1-\frac{\alpha}{2}}\} \rightarrow 1 - \alpha . \quad (5)$$

Invert (5) to obtain a new interval \tilde{I}_n which also satisfies $P\{p \in \tilde{I}_n\} \rightarrow 1 - \alpha$, and write down the interval explicitly. Evaluate both \hat{I}_n and \tilde{I}_n in the case where $n = 100$, $\alpha = 0.05$, and $X = 60$.