Statistics 200, Homework 3

Due Thursday, January 31, 2008

The first five problems are from the text: 8.2, 8.4, 8.7, 8.13, 8.23.

6. Suppose $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{P} 0$. Show that $X_n + Y_n \xrightarrow{d} X$. Note: you are to do this problem using the basic definitions, and not using results from lecture. I sketched the proof in class, and you are asked to complete the exercise.

7. Let X be binomial based on n trials and success probability p. Let $\hat{p}_n = X/n$. Also, let $z_{1-\alpha}$ denote the $1-\alpha$ quantile of the standard normal distribution $N(0,1)$. From class we know that

$$\frac{n^{1/2}(\hat{p}_n - p)}{[\hat{p}_n(1-\hat{p}_n)]^{1/2}} \xrightarrow{d} N(0,1) \ , \tag{1}$$

which leads to the confidence interval statement that

$$P\{p \in \hat{I}_n\} \to 1 - \alpha \ , \tag{2}$$

where $\hat{I}_n$ is the interval

$$\hat{I}_n = \hat{p}_n \pm z_{1-\frac{\alpha}{2}} n^{-1/2} [\hat{p}_n(1-\hat{p}_n)]^{1/2} \ . \tag{3}$$

Instead of this interval, first start with the approximation obtained from the limit result

$$\frac{n^{1/2}(\hat{p}_n - p)}{[p(1-p)]^{1/2}} \xrightarrow{d} N(0,1) \ , \tag{4}$$

which implies

$$P\{-z_{1-\frac{\alpha}{2}} \le \frac{n^{1/2}(\hat{p}_n - p)}{[p(1-p)]^{1/2}} \le z_{1-\frac{\alpha}{2}}\} \to 1 - \alpha \ . \tag{5}$$

Invert (5) to obtain a new interval $\tilde{I}_n$ which also satisfies $P\{p \in \tilde{I}_n\} \to 1-\alpha$, and write down the interval explicitly. Evaluate both $\hat{I}_n$ and $\tilde{I}_n$ in the case where $n = 100$, $\alpha = 0.05$, and $X = 60$.