Statistics 217, Homework 2, Due April 19, 2003

1. Let $u_{i,N}$ denote the probability that a gamber with i dollars is ruined playing against an adversary with $N-i$ dollars, assume the gambler wins or loses one dollar on each (independent) play with probability p. From the handout, we know:

$$u_{i,N} = \frac{(q/p)^i - (q/p)^N}{1-(q/p)^N}, \quad if\ p \neq q$$

and

$$u_{i,N} = 1 - \frac{i}{N}, \quad if\ p = q.$$

In terms of the $u_{i,N}$, write expressions for the following.
1a. In a p up q down random walk starting at the origin, what is the probability of reaching the point $a > 0$ before returning to the origin?
1b. Starting at $a > 0$, what is the probability of reaching the origin before returning to the starting point?

2. Again consider a random walk starting at the origin. Assume $p = 1/2$. What is the distribution of the number of visits to the point a before returning to the origin?

3. A clinical trial is run to determine which of two drugs is more effective in treating a particular disease. Independently, as pairs of patients enter the trial, one randomly gets drug A and the other gets drug B. The outcome is (A_i, B_i) where $A_i = 1$ if drug A cured the patient taking drug A, 0 if not. $B_i = 1$ if drug B cured the other patient. Let

$$S_n = \sum_{i=1}^{n} (A_i - B_i)$$

denote the net difference in the number of cures out of n pairs, so that $S_n > 0$ means drug A is doing better than B. Suppose the trial is run until the first trial n such that $|Z_n| = M$, where M is some fixed number. If M is large, this seems to be a good way of deciding which drug is better. But it is still possible to make an error. Let $a = P\{A_i = 1\}$ and $b = P\{B_i = 1\}$ denote the unknown *cure* probabilities, and assume the A_i and B_i form independent sequences, each being an i.i.d. Bernoulli sequence with success probabilities a and b, respectively.
(i) Let

$$p = P\{A_i - B_i = 1 | A_i - B_i \neq 0\}\ .$$

In terms of a and b, what is p?
(ii) Assuming $a > b$, let α_M be the chance the trial stops by declaring B to be better, the *wrong* conclusion. What is α_M in terms of p and M? (Your solution should clearly show α_M can be made arbitrarily small by choosing M large.)

4. Problem 6.1.

5. Problem 6.4.

6. Problem 6.6.