

Does it work?

Path Analysis and Encouragement Design

Stat 209 4/19

drv summary

Encouragement Designs: Effects of Interventions

Exemplar Study: Random assignment of students to treatment-control conditions for intervention on improving study habits. **Measures:** Treatment/control assignment (G), amount of study (R), and outcome measure, achievement test score (Y).

Questions: 1. Increase in study time from intervention? 2. Increase in achievement from studying an hour longer (dose response)? 3. Increase in achievement if no increase in study (placebo effect)? 4. Total impact on achievement?

Counterfactual Data Formulation for Individual u. 1. $R_t(u) - R_c(u) = \rho(u)$, treatment/control difference in amount of study. 2. $Y_{Gr}(u) - Y_{Cr}(u) = \beta(u) * (r - r')$, increment to outcome from study amount r' vs r . 3. $Y_{tr}(u) - Y_{cr}(u) = \tau(u)$, treatment/control difference in outcome with same amount of study r . 4. $Y_{tR}(u) - Y_{cR}(u) = \tau(u) + \rho(u)\beta(u)$, overall treatment/control difference.

Path Analysis Regressions
Path Coefficients: $\gamma_1, \gamma_2, \gamma_3$

$$R = \alpha_R + \gamma_1 G + \epsilon_R$$

$$Y = \alpha_Y + \gamma_2 R + \gamma_3 G + \epsilon_Y$$

Individual Level Model. $R_G(u) = R_c(u) + \rho(u)G$; $Y_{Gr}(u) = Y_{cR}(u) + \tau(u)G + \beta(u)r$.

ALICE specification: $\rho(u) = \rho$; $\beta(u) = \beta$; $\tau(u) = \tau$.

Path Analysis Results Under ALICE. $\gamma_1 = \rho$; $\gamma_2 = \beta + \delta$; $\gamma_3 = \tau - \rho\delta$.

Indirect/Direct Effects: $\beta\rho/\tau$ under ALICE; $\rho(\beta + \delta)/(\tau - \rho\delta)$ from path analysis.

Holland (1988) p. 469

$$\mu_c(r) = \gamma + \delta r. \tag{51}$$

A positive δ means that the more a student would study when not encouraged, the *higher* he or she would score on the test without studying and without encouragement. A negative δ means that the more a student would study when not encouraged, the *lower* he or she would score without studying and without encouragement.

The quantities computed in path analysis are the conditional expectations

$$E(R_S | S) = E(R_c) + \rho S \tag{52}$$

and

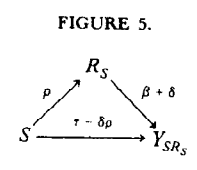
$$E(Y_{SR_S} | S, R_S) = \mu_c(R_S - \rho S) + \tau S + \beta R_S, \tag{53}$$

in which S is a 1/0 indicator variable. If we make the untestable assumption that $\mu_c(r)$ is linear, e.g. (51), then (53) becomes

$$E(Y_{SR_S} | S, R_S) = \gamma + (\tau - \delta\rho)S + (\beta + \delta)R_S. \tag{54}$$

Equations (52) and (54) are both linear and may be combined into the empirical path diagram in Figure 5.

Comparing Figures 5 and 4, we see that even if the ALICE model holds and $\mu_c(r)$ is linear, the estimated path coefficients are



$S = 0/1 (G)$

meaning of δ
 individual differences
 in outcome it
 will R, G

Path Analysis II

4/19

Longitudinal Data (Goldstein et al)

Does it work?

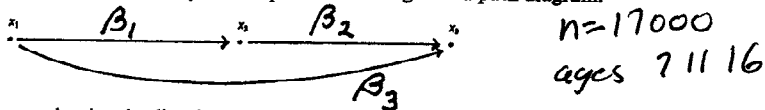
1979

GOLDSTEIN - *Analysing Longitudinal Data*

411

5. MODELS

The first and simplest model to be analysed is represented in Fig. 2 as a path diagram.



$n=17000$
ages 7 11 16

FIG. 2. Path diagram showing the direction of relationships between measurements at three occasions.

Rogosa (1988)

individual-level model straight-line growth in reading

specifies a constant rate of change denoted by θ . The straight-line growth curve for individual p is written:

$$\xi_p(t) = \xi_p(0) + \theta_p t. \quad (1)$$

Path Regressions

Path analysis models for longitudinal data use the temporal ordering of the measurements to delimit the possible paths between the variables. Consider the example of a three-wave design with measures on X at times t_1, t_2, t_3 . The path regressions for the unstandardized variables are:

$$\begin{aligned} X_2 &= \alpha_2 + \beta_1 X_1 + e_2 \\ X_3 &= \alpha_3 + \beta_2 X_2 + \beta_3 X_1 + e_3. \end{aligned} \quad (2)$$

regression (path) coefficients from (2) are:

$$\begin{aligned} \beta_3 &= \frac{t_2 - t_3}{t_2 - t_1} < 0 \\ \beta_2 &= \frac{t_3 - t_1}{t_2 - t_1} > 0. \end{aligned} \quad (3)$$

and $R^2 \approx 1$

data-free causal analysis

[Note: the results in (3) are easily verified by using (1) to substitute $X_2 = X_1 + \theta(t_2 - t_1)$ into the X_3 equation in (2). Substituting the values for β_2 and β_3 from (3) and collecting terms yields $X_3 = X_1 + \theta(t_3 - t_1)$.]

additional results in Trento (dm'93)

3.2. Exponential growth

For a population of individual growth curves given by (2) the population partial regression (structural) coefficients for the η_3 equation in (3) are:

$$\beta_3 = \frac{\exp[-\gamma t_3] - \exp[-\gamma t_2]}{\exp[-\gamma t_1] - \exp[-\gamma t_2]} > 0$$

individ growth

$$\eta_p(t) = \lambda_p - (\lambda_p - \eta_p(0)) e^{-\gamma t}$$

just depends on curvature

$$\beta_2 = \frac{\exp[-\gamma t_3] - \exp[-\gamma t_1]}{\exp[-\gamma t_2] - \exp[-\gamma t_1]} < 0$$

asymptote λ_p curvature γ
linear state dependence $(\gamma) \frac{d\eta}{dt}$