OPERATIONS ON (IN GENERAL) Mean and Variance of a random variable

up on class page

see SW sec 3.7 Adding and subtracting random variables

population mean μ_X population variance σ_X^2

let $Y = a + bX$ (scale-translation)

then $\mu_Y = a + b\mu_X$

$Var(a+bX) = \sigma_{a+bX}^2 = b^2\sigma_X^2$

For 2 random variables X, Z

$E(X+Z) = \mu_{X+Z} = \mu_X + \mu_Z$

$E(X-Z) = \mu_{X-Z} = \mu_X - \mu_Z$

regardless of whether X, Z independent.

Variances: for X, Z independent

$$Var(X+Z) = \sigma^2_{X+Z} = \sigma^2_X + \sigma^2_Z$$

$$Var(X-Z) = \sigma^2_{X-Z} = \sigma^2_X + \sigma^2_Z$$

Sums and Differences of normal r.v. are distributed normal. any linear combination of normals r.v. has normal distribution.

Ex. $X \sim N(\mu_X, \sigma_X)$

$Y \sim N(\mu_Y, \sigma_Y)$

X, Y independent

$$X+Y \sim N\left(\mu_X+\mu_Y, \sqrt{\sigma_X^2+\sigma_Y^2}\right)$$

$$X-Y \sim N\left(\mu_X-\mu_Y, \sqrt{\sigma_X^2+\sigma_Y^2}\right)$$

Ex. Tom and George go golfing

X Tom $X \sim N(110, 10)$

Y George $Y \sim N(100, 8)$

(big variability in scores)

Find $Pr\{\text{George loses to Tom}\}$

$X - Y \sim N(10, 12.8)$

Call $X - Y$ D

```
> pnorm(0,10,12.8)
[1] 0.2173277
```

check using pnorm

$$Pr\{D < 0\} = Pr\left\{\frac{D-10}{12.8} < \frac{-10}{12.8}\right\}$$

$$= Pr\{Z < -.78\} = .2177$$

$$\left[= Pr\{Z > .78\} = .5 - Pr\{0 < Z < .78\} \right]$$