Thresholding non-stationary SPMs with an application to cortical surface mapping

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Introduction

In [1] we have verified and generalized the results of [2] for non-isotropic random fields to the case of random fields on manifolds. In particular, we remove the unverifiable assumption that a "flattening" map exists and approach the problem from a more natural geometric viewpoint. We apply the results to SPM's arising from the analysis of a single subject fMRI trial using the model in [3], restricting the fMRI data to the cortical surface. We use diffusion smoothing on the cortical surface [4] to get an improved estimate of the AR parameter in the fMRI times series model.

Methods

Model and Assumptions

In [5] it was shown that, for isotropic random fields there is an intimate link between the geometry of the search region S and the exceedence probability of the random field f. In particular, for isotropic random fields on $\mathbb{R}^k$

$$\mathrm{P}\left[\sup_{s\in S} f(s) > u\right] \simeq \sum_{j=0}^{k} c_{j,k}\rho_{j,f}(u)\mathcal{M}_{k-j}(S), \tag{1}$$

where $\mathcal{M}_j(S)$ are the called the Minkowski functionals of S and $\rho_{j,f}$ are the Euler characteristic densities of f. This idea was extended in [2] to the case in which the random field is the restriction of an isotropic field in a higher dimensional space. However, there are many random fields where such a mapping does not exist, for instance so-called "scale space" random fields [6].

A more general approach is to remove the assumption of isotropy and look instead at a geometry intrinsically associated to the random field. The Minkowski functionals depend on the curvature of S and, hence, on the geometry of the search region in $\mathbb{R}^k$. This means that they are not invariant under transformations of the parameter space of the random field. Figure 1 below gives a clear example of how the curvature of S in $\mathbb{R}^k$ is not preserved under these transformations.

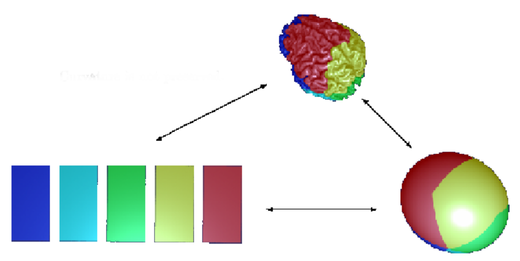


Figure 1: Neither Euclidean distances between points nor curvatures are preserved under transformations of the parameter space.

The distribution of the random field, however, is preserved under these transformations, as shown in Figure 2, and thus, it is natural to look for distances or curvatures of the search region that are invariant under these transformations.

The structure of the random field defines natural distances on S given by the L^p distances between the random variables of the random field, i.e.

$$d_p(s_1, s_2) = \left(\mathrm{E}\left[\left(f(s_1) - f(s_2)\right)^p\right]\right)^{1/p}.$$

If we set $p = 2$, then under certain conditions, the random field f induces a Riemannian metric on S, given by

$$g\left(\left.\frac{\partial}{\partial s_i}\right|_s, \left.\frac{\partial}{\partial s_j}\right|_s\right) = \mathrm{E}\left[\frac{\partial f(s)}{\partial s_i}\frac{\partial f(s)}{\partial s_j}\right].$$

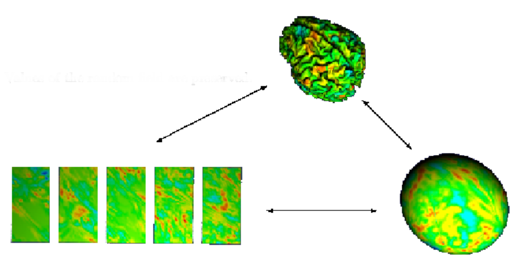


Figure 2: The values of the random are preserved under transformation, unlike the Minkowski functionals.

Thus, every smooth random field induces a Riemannian structure on S and hence turns the pair (S, g) into a Riemannian manifold. What is noteworthy about this is that Riemannian manifolds have intrinsically defined curvatures $\mathcal{L}_j(S)$ called the Lipschitz-Killing curvatures which are thus candidates to replace $\mathcal{M}_j(S)$ in (1). In [1], we have shown, that the following approximation holds for ***any*** smooth Gaussian, χ^2, t or F field

$$\mathrm{P}\left[\sup_{s\in S} f(s) > u\right] \simeq \sum_{j=0}^{k} \frac{1}{(2\pi)^{(n-j)/2}}\rho_{j,f}(u)\mathcal{L}_j(S). \tag{2}$$

Estimating the Lipschitz-Killing curvatures

In practice, we do not know the true covariance function, thus we do not know the Riemannian structure g and in order to apply (2), we must thus find estimates $\hat{\mathcal{L}}_j(S)$. One approach is to fit some sort of functional model to estimate g, for instance when dealing with 2-dimensional surfaces the parametric approach of [4] could be used. Another approach, perhaps better suited to the data is to use some sort of discrete analog of $\mathcal{L}_j(S)$ and thus find a discrete estimator $\mathcal{L}_j(S)$.

As our goal was to analyze data on the cortical surface, already represented as a triangulated surface, we decided to use the discrete Lipschitz-Killing curvatures of [7]. For a triangulation $S^\flat$ of S, the discrete curvatures are defined by:

$$\mathcal{L}_j(S^\flat) = \sum_{\sigma\in S^\flat:\dim(\sigma)=j}\left(\sum_{\sigma'\supseteq\sigma}(-1)^{\dim(\sigma')-j}\beta(\sigma,\sigma')\right)|\sigma|,$$

where β are called the ***external angles*** of the simplex σ in σ'. Since the combinatorial structure of $S^\flat$ is non-random and known from the beginning, the only unknown quantities to be estimated are the external angles. Further, these are functions of the edge lengths of $S^\flat$ only, so any assignment of edge lengths gives an estimate of the Lipschitz-Killing curvatures of S. Following [2] we chose the edge lengths to be the distance between residuals from some linear model, in this case from the linear model of the fMRI time series described in [3].

Application to Cortical Surface Mapping

We applied the method to single subject fMRI data from a pain experiment. The experiment was a block design with two conditions.

Fitting the model for the fMRI time series

The fMRI data was restricted to the triangulated cortical surface by interpolation. The model for the time series, taken from [3] is as follows. The signal is modelled as

$$Y(t,s) = X(t,s)\beta(s) + \varepsilon(t,s), \tag{3}$$

where X is the design matrix, and, for each spatial location s, the time series is supposed to be an AR(1) Gaussian process.

The coefficients β are estimated as follows:

- First we estimate $\hat{\beta}_{LS}$ the ordinary least squares estimates of β.
- Using $\hat{\beta}_{LS}$ calculate residuals $\hat{r}_{LS,i}$ and use these to estimate the AR(1) parameter $\hat{\rho}$.
- After a complete pass through the data, the resulting $\hat{\rho}$ image is smoothed to get $\tilde{\rho}$ using the diffusion smoothing methods found in [4]. In particular, for a fixed smoothing time t_0 (equivalent to FWHM in Gaussian kernel smoothing), the smoothed image

  $$\tilde{\rho} = e^{t_0\Delta}\hat{\rho},$$

  the solution to the heat equation (at time t_0)

  $$\frac{\partial f}{\partial t} - \Delta f = 0, \qquad f(s,0) = \hat{\rho}$$

  with Δ the Laplace-Beltrami operator of S acting on $L^2(S)$ with, in this case, the metric induced by $\mathbb{R}^3$.
- Use the smoothed AR(1) estimate $\tilde{\rho}$ to get $\hat{\beta}_{GLS}$, the generalized least square solution to (3) when $\rho = \tilde{\rho}$.

The resulting $\hat{\beta}$ image for the difference between the two conditions is shown in Figure 3 i).

Choosing the threshold

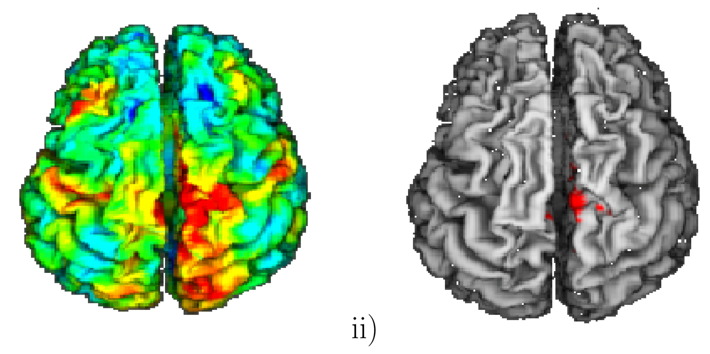
For a triangulation $S^\flat$ of the cortex, a two-dimensional surface S without boundary, the discrete Lipschitz-Killing curvatures are given by:

- $\mathcal{L}_0(S^\flat) = 2\pi\chi(S^\flat) = 4\pi$ since the representation of the cortical surface is topologically equivalent to a sphere,
- $\mathcal{L}_1(S^\flat) = 0$ since this is a boundary term,
- $\mathcal{L}_2(S^\flat) = \mathrm{Area}(S^\flat) \simeq 65,000$, the surface area of the cortex measured in "residual" space (see [2]).

Using these curvatures and the Euler characteristic densities of a Gaussian field given by

$$\rho_{j,f}(u) = \frac{1}{(2\pi)^{(j+1)/2}}H_{j-1}(u)e^{-u^2/2},$$

where H_j is the j-the Hermite polynomial (see [1], for example), we arrive at a $p = 0.05$ threshold of about 5.1. The resulting thresholded $\hat{\beta}$ image is given in Figure 3 ii).



i) ii)

Figure 3: i) Estimated $\hat{\beta}$ image for difference between conditions. ii) $\hat{\beta}$ image thresholded at 5.1, the $p = 0.05$ threshold.

Discussion and Conclusion

In this work we described an intrinsic geometry associated to a smooth random field and discussed how this is relevant to approximations of the excursion probabilities of the random field in (2). As an illustration of the method, we applied it to single subject fMRI data from a pain experiment. While the threshold chosen is very close to that chosen from the assumption of isotropy, the theory described above can also be expected to work in situations where the random fields are very far from isotropic.

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