

STANFORD PROBABILITY SEMINAR

Yuval Peres, Berkeley

Monday, 10 April 2006

4:15pm (Refreshments at 4pm in the 1st Floor Lounge)

Sequoia Hall, Room 200

Hex, Random Turn Games, and the Infinity Laplacian

Abstract. The infinity Laplacian (informally, the “second derivative in the gradient direction”) is a simple yet mysterious operator with many applications, in particular to optimal Lipschitz extensions. Classical analysis of this operator is hampered by nonsmoothness of solutions. ”Tug of war” is a two player random turn game played as follows: Given disjoint target sets T_1 and T_2 in the plane, and a token at x , toss a fair coin; the player who wins the coin toss moves the particle up to distance r in the direction of his/her choice. This is repeated until the token reaches a target set T_i ; player i is then declared the winner. Write $u_r(x)$ for the probability that player 1 wins when both players play optimally. We show that as r tends to 0, the functions $u_r(x)$ converge to the infinity harmonic function with boundary conditions 1 on T_1 and 0 on T_2 . Our analysis of tug of war leads yields new estimates, and significant generalizations of several classical results about infinity Laplacians. I will also describe our original motivation for studying random-turn games: A variant of the game of Hex with a conformally-invariant limit.

(Talk based on joint work with Oded Schramm, Scott Sheffield and David Wilson.)