How Many Entries of A Typical Orthogonal Matrix Can Be Approximated By Independent Normals?

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Abstract I will present my solution to the well-known open problem by Diaconis stated as follows: what are the largest orders of p_n and q_n such that Z_n, the $p_n \times q_n$ left upper block of an n by n typical orthogonal matrix $\mathbf{\Gamma}_n$, can be approximated by independent standard normals? This problem is solved by two different approximation methods.

First, we show that the largest order of p_n and q_n are $o(\sqrt{n})$ in the sense of approximation by the variation norm.

Second, suppose $\mathbf{\Gamma}_n = (\gamma_{ij})_{n\times n}$ is generated by $\mathbf{Y}_n = (y_{ij})_{n\times n}$ through the Gram-Schmidt algorithm where $\{y_{ij}; 1 \le i, j \le n\}$ are i.i.d. standard normals. We show that the largest order of $m = m_n$ such that $\epsilon_n(m) := \max_{1\le i\le n, 1\le j\le m} |\sqrt{n}\gamma_{ij} - y_{ij}|$ *goes to zero in probability* is $o(n/\log n)$.

A history from 1914 to 2003 of the problem from Mechanics, Statistics and Imagine Analysis will also be presented.