STANFORD PROBABILITY SEMINAR

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Monday, 6 December 2004
4:15pm (Refreshments at 4pm in the 1st Floor Lounge)
Sequoia Hall, Room 200

Almost all Cayley graphs for the symmetric group have polynomial diameter

Abstract. Given a group G and a set of generators S, the Cayley graph is the graph with vertex set G and all edges $\{x, gx\}$ such that g is in S. A long-standing conjecture says that there exists a polynomial $p(n)$ such that the diameter of any Cayley graph for the symmetric group S_n is at most $p(n)$, regardless of the set of generators used. The best known upper bound is $\exp(O(\sqrt{n \log n}))$.

We prove that the conjecture holds for almost every pair of permutations generating S_n, and hence for almost every set of generators of any size. Previously, only a superpolynomial bound $n^{(\log n)/2}$ was known for almost all generators.

The proof hinges on finding a large set of random permutations whose first cycle lengths are pairwise "nearly independent."

This is joint work with Laszlo Babai.