STANFORD PROBABILITY SEMINAR

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Monday, 24 January 2005
4:15pm (Refreshments at 4pm in the 1st Floor Lounge)
Sequoia Hall, Room 200

Entropy and graph homomorphisms

Abstract. A homomorphism from a graph G to a graph H is a function $f : V(G) \to V(H)$ which preserves adjacency (if $x \sim y$ in G, then $f(x) \sim f(y)$ in H). Questions about independent (stable) sets in G and proper q-colourings of G, among many others, can be framed in terms of graph homomorphisms. Write $\hom(G, H)$ for the number of homomorphisms from G to H.

Using entropy methods, we show that for any H and any N-vertex, d-regular, biparite G, $\hom(G, H)$ is at most $(\hom(K(d, d), H))^{N/2d}$ where $K(d, d)$ is the complete bipartite graph with d vertices in each partition class. This generalizes a result of J. Kahn on independent sets in regular biparite graphs.

We also give a weighted version of this result which can be interpreted as a statement about the partition function of a statistical physics "spin-system".

This represents joint work with Prasad Tetali, Georgia Tech.