STANFORD UNIVERSITY

DEPARTMENT OF STATISTICS

DEPARTMENTAL SEMINAR

4:15 p.m., Tuesday, August 23, 2005
Sequoia Hall Room 200
(Cookies at 3:45 in 1st Floor Lounge)

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University of Bern, Switzerland

Log-concave density estimation

We study nonparametric maximum likelihood estimation of a univariate log-concave probability density. Many parametric models feature log-concavity (at least for certain parameter ranges), e.g. normal, gamma, extreme value, laplace or logistic. Note further that log-concavity entails unimodality.

The maximum likelihood estimator can be constructed to be the solution of a linearly constrained optimization problem. We provide algorithms using interior point and other methods to solve this optimization task.

Some general properties and two entirely different characterizations of the density estimator $\hat{f}$ are derived. It is shown that its rate of convergence with respect to supremum norm on a compact interval T is at least $(\log(n)/n)^{1/3}$ and typically $(\log(n)/n)^{2/5}$.

This entails that the distribution function estimator $\hat{F}(x) := \int_{-\infty}^{x} \hat{f}(t)\, dt$ is essentially equivalent to the empirical cumulative distribution function.

Furthermore, a general property of log-concave densities enables one to define a simple plug-in monotone hazard rate estimator $\hat{\lambda}$.