STANFORD UNIVERSITY

DEPARTMENT OF STATISTICS

SPECIAL DEPARTMENTAL SEMINAR

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Sequoia Hall Rm. 200

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Fundamental Limits of Bayesian Inference: Order Parameters and Phase Transitions for Road Tracking

There is a growing interest in formulating vision problems in terms of Bayesian inference and, in particular, the maximum a posteriori (MAP) estimator. This approach involves putting prior probability distributions, P(X), on the variables X to be inferred and a conditional distribution P(Y—X) for the measurements Y. For example, X could denote the position and configuration of a road in an aerial image and Y can be the aerial image itself (or a filtered version). We observe that these distributions define a probability distribution P(X,Y) on the ensemble of problem instances. In this paper we consider the special case of detecting roads from aerial images (Gemand and Jedynak 1996) and demonstrate that analysis of this ensemble enables us to determine fundamental bounds on the performance of the MAP estimate (independent of the inference algorithm employed).

We demonstrate that performance measures – such as the accuracy of the estimate and whether the road can be detected at all – depend on the probabilities P(Y—X), P(X) only by an order parameter K. Intuitively, K summarizes the strength of local cues (as provided by local edge filters) together with prior information (i.e. the probable shapes of roads). We demonstrate that there is a phase transition at a critical value of the order parameter K – below this phase transition it is impossible to detect the road by any algorithm. In related work (Yuille and Coughlan 1999), we derive closely related order parameters which determine the time and memory complexity of search and the accuracy of the solution using the A* search strategy. Our approach can be applied to other vision problems and we briefly summarize results when the model uses the "wrong prior" . We comment on how our work relates to studies of the complexity of visual search (Tsotsos 1991) and to

critical behaviour (i.e. phase transitions) in the computational cost of solving NP-complete problems (Selman and Kirkpatrick).