

STANFORD UNIVERSITY  
DEPARTMENT OF STATISTICS  
DEPARTMENTAL SEMINAR

1:00 p.m., Friday, June 30, 2000  
Sequoia Hall Rm. 200

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**Circularly-Coupled Markov Chain Sampling**

I show how to run an  $N$ -time-step simulation of a Markov chain that converges to some equilibrium distribution in a circular fashion, so that the state at time 0 follows the state at time  $N-1$  in the same way as states at times  $t$  follow those at times  $t-1$  for  $0 \leq t \leq N$ . This wrap-around of the chain is achieved using a coupling procedure, and produces states that all have close to the equilibrium distribution of the Markov chain, under the assumption that coupled chains are likely to coalesce in less than  $N/2$  iterations. This procedure therefore automatically eliminates the initial portion of the chain that would otherwise need to be discarded to get good estimates of equilibrium averages. The assumption of rapid coalescence can be tested using auxiliary chains started at times spaced between 0 and  $N$ . When multiple processors are available, such auxiliary chains can be simulated in parallel, and then pieced together to give the circularly-coupled chain, in less time than a sequential simulation would have taken, provided that coalescence is indeed rapid. Use of these procedures depends on development of a good coupling scheme. I discuss coupling schemes based on Metropolis and Langevin updates which allow circular coupling to be used for practical problems. In contrast to exact sampling techniques such as coupling from the past, there is no need to also devise a way of keeping track of large sets of states. On the other hand, although the assumptions behind circular coupling can be tested empirically, the results will not provide an absolute guarantee that the points obtained are from exactly the desired distribution.