

Orthonormal Ridgelets and Linear Singularities

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Abstract:

We construct a new orthonormal basis for $L^2(\mathbb{R}^2)$, whose elements are angularly integrated ridge functions — **orthonormal ridgelets**.

The new basis functions are in $L^2(\mathbb{R}^2)$ and so are to be distinguished from the ridge function approximation system called ridgelets by Candès (1997, 1998), as ridge functions are not in $L^2(\mathbb{R}^2)$.

Orthonormal ridgelet expansions have an interesting application in nonlinear approximation: the problem of efficient approximations to objects such as $1_{\{x_1 \cos \theta + x_2 \sin \theta > a\}} e^{-x_1^2 - x_2^2}$ which are smooth away from a discontinuity along a line. The orthonormal ridgelet coefficients of such an object are *sparse*: they belong to every ℓ^p , $p > 0$. This implies that simple thresholding in the ridgelet orthobasis is, in a certain sense, a near-ideal nonlinear approximation scheme.

The ridgelet orthobasis is the isometric image of a special wavelet basis for Radon space; as a consequence, ridgelet analysis is equivalent to a special wavelet analysis in the Radon domain. This means that questions of ridgelet analysis of linear singularities can be answered by wavelet analysis of point singularities. At the heart of our nonlinear approximation result is the study of a certain tempered distribution on \mathbb{R}^2 defined formally by $S(u, v) = |v|^{-1/2} \sigma(u/|v|)$ with σ a certain smooth bounded function; this is singular at $(u, v) = (0, 0)$ and C^∞ elsewhere. The key point is that the analysis of this point singularity by tensor Meyer wavelets yields sparse coefficients at high frequencies; this is reflected in the sparsity of the ridgelet coefficients and the good nonlinear approximation properties of the ridgelet basis.