Conditioned Diffusions

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Abstract:

This work considers a diffusion process $X(s)$, $s \geq 0$, subject to killing at a rate which depends on time and current value of a diffusion. A conditioned process is defined, on $[0,t]$, $t > 0$, by restricting the sample space of $X(s)$, $s \geq 0$, to the sample paths which survived up to time t. It is shown that conditioned process is a diffusion and its infinitesimal parameters are derived in terms of the infinitesimal parameters of $X(s)$, $s \geq 0$, and in terms of the survival function of $X(s)$, $s \geq 0$. This result is applied to the problem of identification of a parametric form of a killing-rate function such that an unconditioned and an associated conditioned diffusion belong to the same family of diffusions. The specific results are obtained for Gaussian and for branching diffusions, a class of continuous space branching processes. For these diffusions the required killing-rate functions are idnetified and the properties of associated diffusions are discussed. In particular formulas for marginal survival functions of a Gaussian diffusion, and of a branching diffusion are obtained. A general formula for the survival function of a diffusion when the values of a diffusion are known at discrete times is also discussed. The applications of this work to survival analysis and possible extensions are indicated.