Title:

Testing Stochastic Processes: Stationarity, Independence and Ergodicity

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Abstract:

Two sets of probability measures H_0 and H_1 are called discernible, when there exists a sequence of discerning functions $f_n : R^n \to \{0, 1\}$ such that almost surely only finitely many errors are made. This paper studies the discernibility of various families of stochastic processes. First, three different modes of discernibility are introduced: discernibility with an entire sample path (DES), uniform discernibility with an entire sample path (UDES), and sequential discernibility (SD). These modes are shown to have a strictly nested structure. Second, using phase transition phenomena in random coin tossing, we construct criteria for SD between stationary (independent) and non-stationary (dependent) measures. The proposed criteria are functions of (i) how much non-stationary components deviate from their stationary counterparts and (ii) how frequently non-stationary components occur in a long sequence. Third, we study the SD between the independent fair coin tossing μ_0 and the sparse heterogeneous mixtures $\mathrm{HM}(\gamma, \theta) := (1 - \epsilon_n)\mathrm{Bernoulli}(1/2) + \epsilon_n \mathrm{Bernoulli}((1 + \theta)/2)$, where $\epsilon_n = n^{-\gamma}$ with $\gamma \in (0, 1)$. It is shown that $\mathrm{HM}(\gamma, \theta)$ is sequentially discernible from μ_0 when $\gamma \in (0, 0.5)$, but it is not when $\gamma \in (0.5, 1)$. We also give a negative result for testing ergodicity and a general criterion on SD for a broader class of stochastic processes.