Title:
Near-Optimal Detection of Geometric Objects by Fast Multiscale Methods

Author(s):
Ery Arias-Castro, David Donoho and Xiaoming Huo

Technical Report number (Dept. of Statistics, Stanford Univ.):
2003-22

Date:
August 2003

Abstract:

We construct detectors for 'geometric' objects in noisy data. Examples include a detector for presence of a line segment of unknown length, position, and orientation in two-dimensional image data with additive Gaussian white noise. We focus on two issues:

(i) *The optimal detection threshold* – i.e. the signal strength below which no method of detection can be successful for large dataset size n.

(ii) *The optimal computational complexity* of a near-optimal detector, i.e. the complexity required to detect signals slightly exceeding the detection threshold.

We describe a general approach to such problems which covers several classes of geometrically-defined signals; for example, with 1-dimensional data, signals having elevated mean on an interval, and, in d-dimensional data, signals with elevated mean on a rectangle, a ball, or an ellipsoid.

In all these problems, we show that a naive or straightforward approach leads to detector thresholds and algorithms which are asymptotically far away from optimal. At the same time, a multiscale geometric analysis of these classes of objects allows us to derive asymptotically optimal detection thresholds and fast algorithms for near-optimal detectors.