

Title:

**On Nonparametric Testing, the Uniform Behavior of the  $t$ -test, and Related Problems**

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Abstract:

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In this article, we discuss some nonparametric hypothesis testing problems with special emphasis on inference for the mean. First, we recall the classical result of Bahadur and Savage (1956) that delineates the impossibility of constructing useful tests of the value of a mean. Using their ideas, we present some simple results showing how their nonexistence results extend to other testing problems, and we answer a conjecture of theirs. Other examples considered are testing whether or not the mean is rational, testing goodness of fit, and equivalence testing. Next, we discuss the uniform behavior of the classical  $t$ -test. For most nonparametric models, the size of the  $t$ -test is one for every sample size. Even if we restrict attention to the family of symmetric distributions supported on a fixed compact set, the  $t$ -test is not uniformly asymptotically level  $\alpha$ . However, the convergence of the rejection probability is established uniformly over a large family with a very weak uniform integrability type of condition. Furthermore, under such a restriction, the  $t$ -test is seen to possess an asymptotic maximin optimality property. We also discuss some other methods that generate hypothesis tests of valid size in finite samples. Some of the results are classical in nature, but they do not appear to be accessible in the literature.