

Title:

**Fastest Mixing Markov Chain on A Graph**

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Abstract:

We consider a symmetric random walk on a connected graph, where each edge is labeled with the probability of transition between the two adjacent vertices. The associated Markov chain has a uniform equilibrium distribution; the rate of convergence to this distribution, i.e., the mixing rate of the Markov chain, is determined by the second largest (in magnitude) eigenvalue of the transition matrix. In this paper we address the problem of assigning probabilities to the edges of the graph in such a way as to minimize the second largest magnitude eigenvalue, i.e., the problem of finding the fastest mixing Markov chain on the graph.

We show that this problem can be formulated as a convex optimization problem, which can in turn be expressed as a semidefinite program (SDP). This allows us to easily compute the (globally) fastest mixing Markov chain for any graph with a modest number of edges (say, 1000), using standard SDP methods. Larger problems can be solved by exploiting various types of symmetry and structure in the problem, and far larger problems (say 100000 edges) can be solved using a subgradient method we describe. We compare the fastest mixing Markov chain to those obtained using two commonly used heuristics: the maximum-degree method, and the Metropolis-Hastings algorithm. For many of the examples considered, the fastest mixing Markov chain is substantially faster than those obtained using these heuristic methods.

We derive the Lagrange dual of the fastest mixing Markov chain problem, which gives a sophisticated method for obtaining (arbitrarily good) bounds on the optimal mixing rate, as well the optimality conditions. Finally, we describe various extensions of the method, including a solution of the problem of finding the fastest mixing reversible Markov chain, on a fixed graph, with a given equilibrium distribution.