

Title: **ASYMPTOTIC EXPANSIONS IN MULTIDIMENSIONAL
MARKOV RENEWAL THEORY AND FIRST PASSAGE TIMES
FOR MARKOV RANDOM WALKS**

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Technical Report number (Dept. of Statistics, Stanford Univ.):
2001-17

Date: **May, 2001**

Abstract:

We prove a d -dimensional renewal theorem, with an estimate on the rate of convergence, for Markov random walks. This result is applied to a variety of boundary crossing problems for a Markov random walk $\{(X_n, S_n), n \geq 0\}$, in which X_n takes values in a general state space and S_n takes values in \mathbf{R}^d . In particular, for the case $d = 1$, we use this result to derive an asymptotic formula for the variance of the first passage time when S_n exceeds a high threshold b , generalizing Smith's classical formula in the case of i.i.d. positive increments for S_n . For $d > 1$, we apply this result to derive an asymptotic expansion of the distribution of (X_T, S_T) , where $T = \inf\{n : S_{n,1} > b\}$ and $S_{n,1}$ denotes the first component of S_n .