Title:

Thick Points for Planar Brownian Motion and the Erdos-Taylor Conjecture on Random Walk

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Abstract:

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Let $\mathcal{T}(x,r)$ denote the occupation measure of the disc of radius r centered at x by planar Brownian motion run till time 1. We prove that $\sup_{|x|\leq 1} \mathcal{T}(x,r)/(r^2|\log r|^2) \to 2$ a.s. as $r \to 0$, thus solving a problem posed by Perkins and Taylor (1987). Furthermore, we show that for any $a < 2$, the Hausdorff dimension of the set of "perfectly thick points" x for which $\lim_{r\to 0} \mathcal{T}(x,r)/(r^2|\log r|^2) = a$, is almost surely $2 - a$; this is the correct scaling to obtain a nondegenerate "multifractal spectrum" for Brownian occupation measure in the plane. The proofs rely on a 'multiscale refinement' of the second moment method. As a consequence of our results on Brownian motion, we prove a conjecture about simple random walk in $\mathbf{Z}^2$ due to Erdős and Taylor (1960): The number of visits to the most frequently visited lattice site in the first n steps of the walk, is asymptotic to $(\log n)^2/\pi$. We also determine the corresponding "discrete multifractal spectrum": For $0 < \alpha < 1/\pi$, the number of points visited more than $\alpha(\log n)^2$ times in the first n steps of the walk, is $n^{1-\alpha\pi+o(1)}$.