Title:
Thick Points for Intersections of Planar Sample Paths

Author(s):
Amir Dembo, Yuval Peres, Jay Rosen and Ofer Zeitouni

Technical Report number (Dept. of Statistics, Stanford Univ.):
2001-13

Date:
May, 2001

Abstract:

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Let $L_n^X(x)$ denote the number of visits to $x \in \mathbf{Z}^2$ of the simple planar random walk X, up till step n. Let X' be another simple planar random walk independent of X. We show that for any $0 < b < 1/(2\pi)$, there are $n^{1-2\pi b+o(1)}$ points $x \in \mathbf{Z}^2$ for which $L_n^X(x) L_n^{X'}(x) \geq b^2 (\log n)^4$. This is the discrete counterpart of our main result, that for any $a < 1$, the Hausdorff dimension of the set of *thick intersection points* x for which $\limsup_{r \to 0} \mathcal{I}(x,r)/(r^2 |\log r|^4) = a^2$, is almost surely $2 - 2a$. Here $\mathcal{I}(x,r)$ is the projected intersection local time measure of the disc of radius r centered at x for two independent planar Brownian motions run till time 1. The proofs rely on a 'multi-scale refinement' of the second moment method. In addition, we also consider analogous problems where we replace one of the Brownian motions by a transient stable process, or replace the disc of radius r centered at x by $x + rK$ for general sets K.