Title:
Explicit Nonparametric Confidence Intervals for the Variance with Guaranteed Coverage

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Abstract:

In this paper, we provide a method for constructing confidence intervals for the variance that exhibit guaranteed coverage probability for any sample size, uniformly over a wide class of probability distributions. In contrast, standard methods achieve guaranteed coverage only in the limit for a fixed distribution or for any sample size over a very restrictive (parametric) class of probability distributions. Of course, it is impossible to construct effective confidence intervals for the variance without some restriction, due to a result of Bahadur and Savage (1956). However, it is possible if the observations lie in a fixed compact set. We also consider the case of lower confidence bounds without any support restriction. Our method is based on the behavior of the variance over distributions that lie within a Kolmogorov-Smirnov confidence band for the underlying distribution. The method is a generalization of an idea of Anderson (1967), who considered only the case of the mean; it applies to very general parameters, and particularly the variance. While typically it is not clear how to compute these intervals explicitly, for the special case of the variance we provide an algorithm to do so. Asymptotically, the length of the intervals is of order $n^{-1/2}$ (in probability), so that, while providing guaranteed coverage, they are not overly conservative. A small simulation study examines the finite sample behavior of the proposed intervals.